Tutorial-2
Low Noise Amplifier (LNA) Design

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Objective:
Low noise amplifiers are one of the basic building blocks of any communication system. The purpose of the LNA is to amplify the received signal to acceptable levels with minimum self generated additional noise. Gain, NF, non-linearity and impedance matching are four most important parameters in LNA design. The objective of this tutorial is to outline the basic tradeoffs between different amplifying topologies w.r.t gain, NF and impedance matching. After this comparison it is concluded that inductor degenerated common source topology gives the best performance to meet the gain, NF, and impedance matching goals with minimum power consumption in case of narrow band designs.

Goals:
After this tutorial, students should be able to

- Calculate the gain, input impedance and NF of common gate, common source, and shunt feedback amplifiers.
- Understand the basic equations and tradeoff between different LNA topologies.
- Perform the calculation for inductor degenerated common source topology and understand the tradeoff between the gain, NF, and impedance matching.

A supplement tutorial LNA lab is also part of this course which takes the circuit from Problem-2.8 and guides through different analysis to design and practical LNA.
Problem-2.1(Tutorial)

\( \omega_T \) is single figure of merit for high frequency transistors. This is defined as frequency at which current gain is extrapolated to fall to unity.
Although the dc gate current of an MOS transistor is essentially zero, the high- frequency behavior of the transistor is controlled by the capacitive elements in the small- signal model, which cause the gate current to increase as frequency increases.

a) Derive the expression for \( \omega_T \).

b) For RF design we always use minimum length transistors. Why?

Solution:

a).

\[
G = V_{gs}^+ - V_{gs}^- \]

\[I_{in} \quad I_o \]

\[\begin{align*}
V_{gs}^- & \quad G \quad C_{gd} \quad D \quad I_o \\
S & \quad C_{gs} + C_{gb} \quad g_m V_{gs} \\
\end{align*}\]

\[V_{ab} = V_{ds} = 0\]

So \( g_{ms}, r_o, C_{sb}, C_{db} \) have no effect on calculations. (This is drawback of \( \omega_T \) definition)

\[i_i = j\omega (C_{gs} + C_{gb} + C_{gd})V_{gs}\]

\[i_i \approx g_m V_{gs}\]

\[\frac{i_o}{i_{in}} = \frac{g_m}{j\omega (C_{gs} + C_{gb} + C_{gd})}\]

According to definition \( \frac{i_o}{i_{in}} = 1 \) at \( \omega_T \)

\[\omega = \omega_T = \frac{g_m}{C_{gs} + C_{gb} + C_{gd}}\]

\( C_{gb} \) and \( C_{gd} \) are small compared to \( C_{gs} \)

So, \( \omega_T \approx \frac{g_m}{C_{gs} + C_{gd}} \approx \frac{g_m}{C_{gs}} \)  

\[I_D = \frac{\mu_o C_{ox} W}{2L} (V_{gs} - V_T)^2\]

\[g_m = \frac{\partial I_D}{\partial V_{gs}} = \mu_o C_{ox} \frac{W}{L} (V_{gs} - V_T)\]
And $C_{gs} = C_o \times WL$  

Put (2) & (3) in (1)

$$\omega_T = \frac{g_m}{C_{gs}} = \frac{\mu_o C_o W (V_{gs} - V_t)}{L C_{ox} W L} = \frac{\mu_o (V_{gs} - V_t)}{L^2}$$

$$\omega_T = \frac{\mu_o (V_{gs} - V_t)}{L^2}$$

This means that $\omega_T \propto \frac{1}{L^2}$ so that’s why minimum $L$ is preferred. But this approximation holds for long channel devices for short channel $\omega_T \propto \frac{1}{L}$ instead of $\frac{1}{L^2}$.

**Problem-2.2(Tutorial)**

NMOS transistor is racing horse in LNA design arena due to its higher mobility compared to PMOS transistors. Calculate the IP3 of NMOS CS amplifier shown below. Assume that NMOS transistor is in saturation.

![NMOS CS Amplifier Diagram](image)

a) Consider simplified square law model. (HW)

$$I_D = \frac{K_n}{2} (V_{gs} - V_T)^2$$

b) Consider the short channel effects as  (Tutorial)

$$I_D = \frac{K_n}{2} \left[ \frac{(V_{gs} - V_T)^2}{1 + \theta(V_{gs} - V_T)} \right]$$

$$\theta = Velocity\ Saturation,\ Mobility\ Degradation$$

$$V_{gs} - V_T = 0.2V \quad and \quad \theta = 0.1W^{-1}$$

c) What conclusion can be drawn from part b) about the bias current and transconductance of transistor for higher IP3?

**Solution:**
a). Home work Ans: \( IP3 = \alpha \)

b). From Razavi

\[
y(x) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) \quad \text{---------}(1)
\]

\[
x(t) = A \cos \omega t + A \cos \omega_2 t \Rightarrow IP3 = \sqrt{\frac{4}{3}} \left| \frac{\alpha_1}{\alpha_3} \right|
\]

\[
I_D = \frac{K_n}{2} \frac{(V_{GS} - V_T)^2}{1 + \theta(V_{GS} - V_T)}
\]

Here we assume that small signal \( x(t) \) over-rides \( (V_{GS} - V_T) \).

So,

\[
I_D = \frac{K_n}{2} \frac{[(V_{GS} - V_T) + x(t)]^2}{1 + \theta(V_{GS} - V_T + x(t))}
\]

\& \( V_{GS} - V_T = \Delta V \) \quad \text{--------- Large signal}

X(t) \quad \text{--------- Small signal}

\[
I_D = \frac{K_n}{2} \frac{[x(t) + \Delta V]^2}{\theta(x(t) + \Delta V) + 1}
\]

\[
V_o = I_D R_L \Rightarrow V_o = \frac{K_n R_L}{2} \frac{(x(t) + \Delta V)^2}{1 + \theta(x(t) + \Delta V)} \quad \text{put} \quad \frac{K_n R_L}{2} = K
\]

\[
\theta << 1 \quad \text{So} \quad (x(x) + \Delta V) \quad \text{is also small} \Rightarrow \frac{1}{1 + x} = 1 - \frac{x}{2}
\]

\[
\frac{1}{1 + \theta(x(t) + \Delta V)} \approx 1 - \frac{\theta(x(t) + \Delta V)}{2}
\]

\[
V_o = K(x(t) + \Delta V)^2 \left(1 - \frac{\theta(x(t) + \Delta V)}{2}\right)
\]

\[
V_o = K(x(t) + \Delta V)^2 - (x(t) + \Delta V)^3 \frac{K\theta}{2}
\]

\[
V_o = K\Delta V^2 - \frac{K\theta}{2} \Delta V^3 + \left(2K\Delta V - \frac{3K\theta}{2} \Delta V^2\right) x(t) + \left(K - \frac{3K\theta}{2} \Delta V\right) x^2(t) - \frac{K\theta}{2} x^3(t)
\]

\text{---------}(2)

Small signal components

\[
V_o = \left(2K\Delta V - \frac{3K\theta}{2} \Delta V^2\right) x(t) + \left(K - \frac{3K\theta}{2} \Delta V\right) x^2(t) - \frac{K\theta}{2} x^3(t) \quad \text{---------}(3)
\]
Comparing (1) & (3)

\[ \alpha_1 = 2K\Delta V - \frac{3K\theta}{2}\Delta V^2, \quad \alpha_2 = K - \frac{3K\theta}{2}\Delta V, \quad \alpha_3 = -\frac{K\theta}{2} \]

\[ IP3 = \frac{4}{3} \frac{\alpha_1}{\alpha_3} = \frac{4}{3} \times \frac{2K\Delta V - \frac{3K\theta}{2}\Delta V^2}{K\theta} = \frac{8}{3} \left( \frac{2\Delta V}{\theta} - 3\Delta V^2 \right) \]

\[ IP3 = \sqrt[3]{\frac{82\Delta V}{\theta}} \]

\[ IP3 = \sqrt[3]{\frac{16\Delta V}{\theta}} \]

\[ IP3 = \sqrt[3]{\frac{16(V_{GS} - V_T)}{\theta}} \]

\[ IP3 = \sqrt[3]{\frac{160.2}{3.1}} = 3.27\text{Volts} \]

\[ IP3(dBm) = 10\log \left[ \left( \frac{3.27}{\sqrt{2}} \right)^2 \cdot \frac{1}{50} \right] \cdot 1mW \approx 20\text{dBm} \]

\[ g_m = \frac{2}{V_{GS} - V_T} \]

\[ I_D = \frac{\mu_n C_{ox} W}{L} \]

\[ I_D = \frac{\partial I_D}{\partial V} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T) \]

\[ g_m = \frac{2}{I_D} \]

\[ V_{GS} - V_T = \Delta V \]

\[ g_m = \sqrt{\mu_n C_{ox} \frac{W}{L} I_D} \]

\[ \Rightarrow IP3 \approx \sqrt{32} \frac{I_D}{g_m} \quad \text{or} \quad g_m = \sqrt{\mu_n C_{ox} \frac{W}{L} I_D} \]

\( \text{c).} \)

- To increase IIP3 \( I_D \uparrow \) (high power) or \( g_m \downarrow \) (high noise)
- \( g_m \) also depends upon \( I_D \), so when \( I_D \uparrow \) \( g_m \uparrow \) but at that rate \( \alpha \sqrt{I_D} \)
- When \( W \) increases \( g_m \uparrow \) for same \( I_D \) (Power consumption) so this decreases IP3
- The above observations are for long-channel. But for short channel \( (4) \Rightarrow (V_{GS} - V_T) \uparrow \) then \( I_D \uparrow \) any how.

**Problem-2.3 (Tutorial)**

It is preferred in current RF designs that the input of LNA be matched to 50 \( \Omega \) (Razavi, Pg168). The easiest way is to shunt the gate with a resistor of 50 \( \Omega \).

a) Calculate the gain, input impedance and NF in absence of gate noise. Assume that \( R_{sh} = R_L \) for NF derivation.

b) What are the disadvantage of shunt resistor with reference to gain and NF?
Solution:

a). (Please read assumption in problem statement carefully)

\[ F = \frac{\text{Total output noise power}}{\text{Output noise due to input source}} \]

\[ \overline{V^2_{m,R_s}} = 4KTR_s \Delta f \]

\[ \text{Gain}_{\text{Gate}} = -g_m R_L \]

\[ \overline{V^2_{m,R_{sh}}} = 4KTR_{sh} \Delta f \]

\[ A = g_m R_L \left( \frac{R_{sh}}{R + R_{sh}} \right) \text{ for } R_{sh} = R_s \]

\[ i_d^2 = 4KT \gamma g_m \Delta f \]

\[ A = -g_m \frac{R_L}{2} \]

Using superposition, considering one at a time and shorting / opening other sources.

\[ \overline{V^2_{on,R_s}} = \overline{V^2_{n,R_s}} \times g_m^2 R_L^2 \times \left( \frac{R_{sh}}{R + R_{sh}} \right)^2 \]

\[ \overline{V^2_{on,R_{sh}}} = \overline{V^2_{n,R_{sh}}} \times g_m^2 R_L^2 \times \left( \frac{R_s}{R + R_{sh}} \right)^2 \]
\[
V_{no,d}^2 = \bar{i}_d^2 \times R_L^2
\]

\[
F = \frac{V_{on,RS}^2 + V_{on,Rsh}^2 + V_{no,d}^2}{V_{on,RS}^2} = 1 + \frac{V_{on,Rsh}^2 + V_{o,d}^2}{V_{on,RS}^2}
\]

\[
F = 1 + \frac{4KTR_{sh} \Delta f \times g_m^2 R_L^2 \times R_{sh}^2}{(R_s + R_{sh})^2} + \frac{4KTR_s \Delta f \times g_m^2 R_L^2 \times R_s^2}{(R_s + R_{sh})^3}
\]

In case of impedance match \(R_s = R_{sh}\)

\[
F = 1 + 1 + \frac{R_L g_m}{R_s \times \frac{g_m^2 R_s^2 \times R_L^2}{4R_s^2}} = 2 + \frac{R_L g_m}{R_s \times \frac{g_m^2 R_s^2 \times R_L^2}{4}} = 2 + \frac{4\gamma}{g_m R_s}
\]

b).

- Poor Noise Figure
- Input signal attenuated by voltage divider
- \(R_{sh}\) adds extra noise.
- At high frequency, shunt \(L\) is needed to tune out \(C_{gs}\)
- Reduced gain

**Problem-2.4 (Tutorial)**

Another approach to get 50 Ω input impedance match is shunt feedback amplifier shown below.

![Shunt Feedback Amplifier Diagram](image)

a) Calculate the gain, input impedance and NF in absence on gate noise. Neglect gate drain and gate to bulk and gate to source capacitance.

b) What are the disadvantage of shunt feedback amplifier with reference to gain and NF?
Solution:

(Equivalent noise model ignoring gate noise), RL is noiseless (Baising not shown)

$$I_{nD}^2 = 4KT\gamma g_m \Delta f, V_{RS}^2 = 4KTR_S \Delta f$$

$$NF = \frac{V_{n,\text{out}}^2}{A_{v,\text{tot}}^2 V_{RS}^2}$$

Total input noise power 
Output noise power due to input source

Here $A_{v,\text{tot}} = \text{Gain from } V_{\text{in}} \text{ to } V_{\text{out}}$

Again using superposition theorem

$$NF = \frac{V_{n,\text{out}}^2}{A_{v,\text{tot}}^2 V_{RS}^2} = \frac{V_{n,\text{out}}^2 + V_{n,\text{RF,\text{out}}}^2 + V_{n,\text{D,\text{out}}}^2}{A_{v,\text{tot}}^2 V_{RS}^2}$$

Gain Calculation

$$V_{\text{in}} = i_{\text{in}} (R_S + R_F) + V_{\text{out}}$$

$$V_{\text{out}} = (i_{\text{in}} - g_m V_{gs}) R_L$$

$$V_{gs} = i_{\text{in}} R_F + V_o$$

$$A_{v,\text{tot}} = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_L (1 - g_m R_L)}{R_S + R_F + R_L + g_m R_S R_L}$$

If $R_F >> R_S$ & $g_m R_F >> 1$

$$A_{v,\text{tot}} = - \frac{g_m R_L}{R_F + 1 + R_L + \frac{1 + R_L}{R_F}} \approx - g_m R_L$$

$$A_{v,\text{tot}} \approx - g_m R_L$$

Also $Z_{\text{in}} = \frac{R_F + R_L}{1 + g_m R_L}$

By ignoring $C_{gs}$, we have considered real part only.
For source resistance:

\[
\overline{V}_{nRS,\text{out}}^2 = A_{v,\text{tot}}^2 \overline{V}_{nRS}^2 \tag{1}
\]

For feedback resistance:

\[
\begin{align*}
V_{gs} &= -iR_s = iR_F - V_{RF,\text{out}} \\
V_{RF,\text{out}} &= R_L (i - g_m V_{gs}) \\
V_{RF,\text{out}} &= V_{RF} \frac{1}{1 + \frac{R_s + R_F}{R_L (1 + g_m R_s)}} = V_{RF} \frac{R_L}{R_F} (1 + g_m R_s) \\
\overline{V}_{n,RF,\text{out}}^2 &= \overline{V}_{n,RF}^2 \left( \frac{R_L}{R_F} (1 + g_m R_s) \right)^2 \tag{2}
\end{align*}
\]

Similarly:

\[
\begin{align*}
\frac{V_{nD,\text{out}}}{R_L} + \frac{I_{nD}}{R_L} + g_m V_{gs} + \frac{V_{nD,\text{out}}}{R_s + R_F} &= 0 \\
V_{gs} &= R_S \frac{V_{nD,\text{out}}}{R_s + R_F} \\
V_{nD,\text{out}} &= \frac{I_{nD}}{R_L} + \frac{1}{R_s + R_F} + \frac{g_m R_S}{R_s + R_F} \approx I_{nD} R_L \\
\overline{V}_{nD,\text{out}}^2 &= \overline{I}_{nD}^2 R_L^2 \tag{3}
\end{align*}
\]

Combining (1), (2), and (3):
\[
NF = 1 + \frac{V_{n,RF}^2}{R_F}\left[\frac{R_L}{R_F}(1 + g_m R_S)\right]^2 + \frac{I_{nD}^2 R_L}{A_{v,tot} V_{n,RS}^2}
\]

\[
A_{v,tot} = -g_m R_L, \quad V_{n,RS}^2 = 4KTR g_m \Delta f, \quad V_{M,RF}^2 = 4KTR_F & I_{nD}^2 = 4KT g_m
\]

\[
NF = 1 + \frac{R_S}{R_F}\left(1 + \frac{1}{g_m R_S}\right)^2 + \frac{\gamma}{g_m R_S}
\]

b).

NF \downarrow g_m R_S \uparrow & R_F \uparrow usually \quad R_S = 50 \Omega

- Better performance than CS amplifier
- \(R_F\) induces noise
- At \(f\) \uparrow need shunt inductor to tune out \(C_{gs}\)
- Broadband Amp @ Lower frequency
- To make NF \(R_F > R_S g_m R_S >> 1\)

Problem-2.5 (HW)

Common gate amplifier also offers 50 \(\Omega\) input impedance match and solves the input matching problem.

c) Calculate the gain, input impedance and NF in absence on gate noise. Neglect gate drain and gate to bulk and gate to source capacitance.

a) What are the disadvantage of common gate amplifier with reference to gain and NF?

Problem-2.6 (Tutorial)

The disadvantages of three types of amplifiers in Problem-3, 4 & 5 can be circumvented by using source degenerated LNA shown below.
a) Calculate the input impedance. This inductor source degenerated amplifier presents a noiseless resistance for 50Ω for input power match. How we can cancel the imaginary part of complex input impedance so that the LNA presents 50Ω real input resistance at input port.

b) Calculate the NF in absence on gate noise. Neglect gate drain and gate to bulk and gate to source capacitance.

c) $C_{gd}$ bridges the input and output ports. The reverse isolation of this LNA is very poor. Why reverse isolation is important? Suggest the modification to improve reverse isolation.

Solution:

a).

From model above we can write

$$V_{in} = i_{in} (j\omega L_g + j\omega L_s) + i_{in} \left( \frac{1}{j\omega C_{gs}} \right) + i_o j\omega L_s \quad \text{-------------(1)}$$

$$i_o = g_m V_{gs} = g_m i_{in} \times \frac{1}{j\omega C_{gs}} \quad \text{-------------(2)}$$

Substituting (2) in (1)

$$V_{in} = i_{in} \left[ j\omega (L_g + L_s) + \frac{1}{j\omega C_{gs}} + \frac{g_m L_s}{C_{gs}} \right]$$
\[ Z_{in} = \frac{V_{in}}{i_{in}} = j\omega(L_g + L_s) + \frac{1}{j\omega C_{gs}} + \frac{g_m L_s}{C_{gs}} \]

\[ Z_{in} = j\omega(L_g + L_s) + \frac{1}{j\omega C_{gs}} + \frac{g_m L_s}{C_{gs}} \]

For matching \( L_g + L_s \) are canceled out by \( C_{gs} \). So at frequency of interest

\[ \omega_o (L_g + L_s) = \frac{1}{\omega_o C_{gs}} \Rightarrow \omega_o^2 = \frac{1}{(L_g + L_s)C_{gs}} \]

And \( R_S = 50\Omega = \frac{g_m}{C_{gs}} \)

**Notes:**

a). \( L_s \) is typically small and may be realized by bond wire.

b). \( L_g \) can be implemented by spiral/external inductor.

b).

From part a)

\[ Z_{in} = j\omega(L_g + L_s) + \frac{1}{j\omega C_{gs}} + \frac{g_m L_s}{C_{gs}} \]

We can draw this circuit as

\[ \begin{align*}
\text{Here} \\
Q_{in} &= \frac{\omega_o (L_g + L_s)}{R_S + \frac{g_m L_s}{C_{gs}}} = \frac{\omega_o (L_g + L_s)}{R_s + W_T L_s} \\
\therefore \omega_T &\approx \frac{g_m}{C_{gs}} \\
Q_{in} &= \frac{1}{\omega_o \left(R_S + \frac{g_m L_s}{C_{gs}}\right)C_{gs}} \quad \text{For match load } R_S = \frac{g_m L_s}{C_{gs}}
\end{align*} \]

**Reference:**

For series RLC Circuit

\[
\begin{align*}
Q_s &= \frac{1}{R \sqrt{C}} = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC} \\
\text{And } V_C &= Q_s V_{in} \\
\omega_T &\approx \frac{g_m}{C_{gs}} \quad \text{Unity gain frequency for current}
\end{align*}
\]
\[ Q_{in} = \frac{1}{2\omega_o R_s C_{gs}} \]

**Gain**

\[ V_{gs} = Q_{in} V_{in} \]

\[ g_m = \frac{V_{out}}{V_{gs}} \]

\[ G_m = \frac{V_{out}}{V_{in}} = \frac{V_{gs} g_m}{V_{in}} = Q_{in} g_m \]

\[ G_m = Q g_m \]

So, \( Gain = -G_m R_L \) or \( -Q_{in} g_m R_L \) & \( G_m = Q_{in} g_m \)

**Noise Figure:**

\[ NF = \frac{Total \ noise \ power \ at \ output}{noise \ power \ at \ output \ due \ to \ input \ source} \]

For this calculation we ignore channel noise.

\[ F = \frac{V^2_{nRS,OUT} + V^2_{nD,OUT}}{V^2_{nRS,OUT}} = 1 + \frac{V^2_{nD,OUT}}{V^2_{nRS,OUT}} \]

\[ V^2_{nD,OUT} = i^2_{n,D} R^2_L \]

\[ V^2_{nRS,OUT} = \frac{V^2_{n,RS} G^2_m R^2_L}{V^2_{n,RS}} = 4KTR_s \Delta f \]

\[ F = 1 + \frac{i^2_{n,D} R^2_L}{V^2_{n,RS} g^2_m R^2_L} = 4KTR_s \]

\[ F = 1 + \frac{\gamma}{g_m R_s Q^2_{in}} \]

**Notes:**

- Very good NF value
- Narrow band matching
- NF ↓ with \( Q^2 \)
- The Q is dependent upon \( L_g + L_s \), \( L_s \) is small so Q depend upon \( L_g \)
C). **Draw Backs**

i).

\[ \omega_o = \frac{1}{L_D C_L} \]

The \( C_L \) can be the input capacitance of mixer or filter.

ii).

Reverse isolation depends upon \( C_{gd} \) to make it better cascade input device

**Problem-2.7 (HW)**

Fill-in the Table below, use the data from Problem-2.4, 2.5, 2.6 and 2.7

<table>
<thead>
<tr>
<th>Type of LNA</th>
<th>( Z_{in} )</th>
<th>Noise Factor</th>
<th>Gain</th>
<th>NF (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shunt Resistor</td>
<td>( R_{sh} )</td>
<td>[ 2 + \frac{4\gamma}{g_m R_S} ]</td>
<td>[ -\frac{g_m R_L}{2} ]</td>
<td></td>
</tr>
<tr>
<td>Common Gate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shunt Feedback</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Source Degenerated</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Calculate the NF for all above amplifiers. Assume \( \gamma = 2 \), \( g_m = 20\text{mS} \), \( R_S = 50\Omega \), \( R_F = 500\Omega \), and \( Q_{in} = 2 \).

b) Which is best topology for Narrow Band LNA design at high frequency?
Problem-2.8 (Tutorial)

**Real Design:** We will design the inductor source degenerated LNA shown in Fig below to meet the specification outlined for IEEE802.11 (b) standard. The first cut approximate values are calculated as a starting point for simulation. In **LAB3: Design of LNA** you will take the same design and modify these component values to meet the specification.

**LNA Specification:**

NF < 2.5 db, Gain > 15dB, IP3 > -5dBm, Centre Frequency = 2.4 GHz

S11 < -20dB, S22 < -10dB, Load Capacitance = 1pF

**Technology Parameters for 0.35μm CMOS:**

\[ L_{\text{eff}} = 0.35 \mu m, \quad \mu_n C_{ox} = 170 \mu A/V^2, \quad C_{ox} = 4.6 mF/m^2, \quad \mu_p C_{ox} = 58 \mu A/V^2, \quad \gamma = 2 \]

![LNA Circuit Diagram]

**Solution:**

\[ \begin{align*}
\mu_n C_{ox} &= 170 \mu A/V^2, \quad \mu_p C_{ox} = 58 \mu A/V^2, 0.35 \mu m \\
CMOS, C_{ox} &= 4.6 mF/m^2, \quad \gamma = 2, f_0 = 2.4GHz \\
(IEEE802.11(b) Standard), \quad \mu_n C_{ox} &= 170 \mu A/V^2 
\end{align*} \]

**Design Parameters**

\[ R_S = 50 \Omega, \quad V_{DD} = 3.3V, \quad C_L = 10pF, \quad NF < 2.5dB \]
Component Description

- **L** – Matches input impedance
- **L_g** – Set the Resonant Frequency \( f_0 = 2.4 \text{ GHz} \)
- **M_3** – Biasing transistor which forms current mirror with \( M_1 \)
- **L_d** – Tuned output increases the gain and also work as band pass filter with \( C_L \)
- **M_2** – Isolate tuned input and tuned output increases reverse isolation, also reduces the effect of Miller capacitance \( C_{gs} \)
- **C_B** – BC Blocking capacitor chosen to have negligible reactance at \( f_0 = 2.4 \text{ GHz} \)
- **R_{BIAS}** – Large enough so that its equivalent current noise is small enough to be ignored. (Don’t consider it as voltage noise source. Why??)

Design Procedure

Size of \( M_1 \):

We will not go for global minimum noise figure as given by two-point noise theory (See lecture on LNA Slide # 10)

\[
G_{opt} = \frac{\alpha \omega C_{gs} \sqrt{\frac{\delta}{5\gamma}}(1 - |C|^2)}{\frac{1}{50\Omega}} \quad \text{---------------(1)}
\]

\[
C_{gs} \approx 4pF \Rightarrow W_{M1} \approx 4\text{mm}!! \quad \text{(not possible)}
\]

Solution:

A & B are from Thomos. H. Lee book (LNA Chapter)

LNA NF will be optimized for given Power

* It will not be best NF globally.

\[
W_{opt} = \frac{1}{3\omega_o L_{eff} C_{ox} R_S}
\]

\[
F_{\text{min},p} = 1 + 2.4 \frac{\gamma}{\alpha} \frac{\omega}{\omega_r} \Rightarrow F_{\text{min},p} = 1 + 5.6 \frac{\omega}{\omega_r} \quad \text{---------------(A)}
\]

From (1) we can derive

\[
F_{\text{min},p} = 1 + \frac{2}{\sqrt{5}} \frac{\omega}{\omega_r} \sqrt{\chi C(1 - |C|^2)} = 1 + 2.3 \frac{\omega}{\omega_r}
\]

\[
F_{\text{min},p} = 1 + 2.3 \frac{\omega}{\omega_r} \quad \text{---------------(B)}
\]

(a) is minimum NF for a given power consumption.

(b) is global minimum noise figure.

The difference is usually 0.5dB to 1dB (no big deal for Lower Power)
Step - 1:
$I_1 = I_2 = 5mA$ (Low Power consumption)

Step - 2:

\[
W_{M1} = \frac{1}{3WL_{eff} C_{ox} R_S}
\]

\[
W_{M1} = \frac{1}{3 \times 0.35 \mu \times 4.6m \times 50 \times \omega_o}
\]

\[
\begin{align*}
R_S &= 50\Omega, C_{ox} = 4.6mF/m^2, \\
\mu_n C_{ox} &= 170 \mu A/V, L_{eff} = 0.35 \mu m, \\
\omega_o &= 2\pi f_o, f_o = 2.4GHz
\end{align*}
\]

\[
W_{M1} = 3.9 \times 10^{-4}
\]

\[
W_{M1} = 3.9 \times 10^{-4} = 390\mu m
\]

Step - 3:

\[
C_{gs1} = \frac{2}{3} W_{M1} L_{eff} C_{ox}
\]

\[
C_{gs1} = \frac{2}{3} \times 390\mu \times 0.35\mu \times 4.6m = 0.41pF
\]

\[
g_{m1} = \sqrt{\frac{2 \mu_n C_{ox} \left( \frac{W}{L} \right)_{M1}}{I_{DM1}}}
\]

\[
g_{m1} = \sqrt{2 \times 170 \mu \times \left( \frac{390}{0.35} \right) \times 5m = 43mA/V}
\]

\[
\omega_T \approx \frac{g_{m1}}{C_{gs1}} = \frac{43mA/V}{0.41pF} = 104G rad/Sec
\]

Assuming \( \gamma = 2 \)

Now \( F_{min} = 1 + 5.6 \frac{\omega_o}{\omega_T} \)

\[
F_{min} = 1 + 5.6 \frac{2\pi 2.4G}{104G} \approx 2.55dB
\]

\( NF \approx 2.55dB \)

It’s very close to what we derive, if the value is higher we can increase \( I_D \) to increase \( \omega_T \) and hence low \( NF \) on expense of power.
Step - 4:
Source and gate inductance such that they cancel $C_{gs}$ and set $50\Omega$ input impedance

\[ \omega_o = 2\pi f_o = 2\pi \times 2.4 = 15\text{G rad/Sec} \]

From previous problem

\[ R_S = R_{\text{Transformed}} = g \frac{L_s}{C_{gs}} = L_s \omega_f \]

\[ L_s = \frac{R_s}{\omega_f} = \frac{50}{100G} \approx 0.5nH \]

$L_s = 0.5nH$ can be implemented using Band wire.

Now $L_g + L_s = \frac{1}{\left(\omega_o C_{gs}ight)}$

\[ L_g + L_s = \frac{1}{\left(15G\right)^2 \times 0.41pF} = 10.81nH \]

$L_g \approx 10nH$

Step - 5:

\[ L_d = \frac{1}{\omega_o^2 C_L} \quad \therefore C_L = 1pF \]

\[ L_d = \frac{1}{\left(15G\right)^2 \times 1pF} \approx 4.4nH \]

$L_d = 4.4nH$

Step - 6:
Size of M3 is chosen to minimize power consumption

\[ W_{M3} = 70\mu m, \quad R_{\text{REF}} = 2K\Omega \Rightarrow I_3 = 0.6mA \]

\[ R_{\text{BIAS}} = 2K\Omega \] (Large enough so that it’s equivalent current noise can be neglected)

\[ C_B = 10pF \quad (X_C \approx 6.6\Omega \text{ so good value @ 2.4G}) \quad X_B = \frac{1}{2\pi f_o C_B} = 6.6\Omega \]

Step - 7:
Size $M2 = M3$
So that they can have shared Drain Area..
(Note: We will simulate same design in LAB # 2)
Problem-2.9 (Point to Ponder):
Connecting two Inductor source degenerated LNA as shown in Figure make the differential LNA. Differential LNA has many advantages: higher common mode rejection ratio, less sensitivity to the ground inductance variation $L_s$ compared to single ended counterpart.

![Differential LNA Circuit Diagram]

a) Compare intuitively the NF of single ended and differential if both have same power consumption.

b) If low power is not parameter on interest, which LNA has lower NF?

Instructions:
For hand calculation of NF you can ignore the gate noise of the device and noise generated by the load resistance $R_L$.

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